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A controlled quantum teleportation scheme of an N-particle unknown state via three-particle W_1 states*

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In this paper a controlled quantum teleportation scheme of an N-particle unknown state is proposed when N groups of three-particle W_1 states are utilized as quantum channels. The quantum information of N-particle unknown state is transmitted from the sender to the recipient under the control of all supervisors. It can be realized with a certain probability. After the sender makes Bell-state measurements and the supervisors perform the computational basis measurements, the recipient will introduce auxiliary particles and carry out unitary transformations depending on classical information from the sender and the supervisors. Finally, the computational basis measurement will be performed by the recipient to confirm whether the teleportation succeeds or not. The successful completion of the scheme relies on all supervisors' cooperation. In addition, the fidelity and security of the scheme are discussed.

Keywords: quantum information, controlled quantum teleportation, W_1 state
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1. Introduction

Quantum teleportation is one of the striking processes in the quantum information theory. It is a communication protocol for transmitting the state of a quantum system from a place to another place without passing the system itself. In 1993, Bennett *et al*[1] first showed that quantum entanglement can be used to teleport an unknown quantum state via an Einstein–Podolsky–Rosen (EPR) pair with the help of some classical information. Teleportation of the polarized photon and a single coherent mode of the radiation field were also realized experimentally.[2,3] Since then, many kinds of schemes of quantum teleportation have been put forward.[4−15]

To date, quantum state sharing, an important branch of quantum information, has attracted a lot of attention.[16−20] One-particle state or two-particle state multiparty sharing protocols are proposed.[16−19] The $n+1$-particle Greenberger–Horne–Zeilinger (GHZ) state which is used as the quantum channel, or general Bell-state measurement (or general GHZ-state measurement) is applied in the process. Recently, the multiparty quantum state sharing of an arbitrary m-qubit state has been studied in Ref.[20]. Quantum state sharing can be used to realize controlled quantum teleportation with or without a little modification, in which the supervisors are included.[21−27] The teleportation can be controlled by the supervisors' sides, and it cannot be completed without all the supervisors' cooperation. In Refs.[17,26,27], the preparation of the quantum channels or the measurements are difficult for the present technique. Sometimes, the sender and the supervisors need to make measurements in turn which takes much time to realize teleportation.[24,25]

The three-particle entangled states were classified as the GHZ class states and the W class states.[28] The W class states are inequivalent to the GHZ class states in the sense that they cannot be converted to each other under stochastic local operations and classical communication. In Ref.[29], the general WGHZ states of three-particle and disentanglement are studied: W_0 state and W_1 state are deduced by means of the general WGHZ states when different disentanglement conditions are obtained. They can be expressed as

$$|W_0\rangle_{ABC} = (a\,|100\rangle + b\,|010\rangle + c\,|001\rangle + d\,|000\rangle)_{ABC}, \tag{1}$$

$$|W_1\rangle_{ABC} = (a\,|100\rangle + b\,|010\rangle + c\,|001\rangle + d\,|111\rangle)_{ABC}, \tag{2}$$

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where the coefficients satisfy normalized condition $|a|^2+|b|^2+|c|^2+|d|^2=1$. W_0 state is applied as quantum channel in Refs.[5, 6, 14], but less attention has been paid to the W_1 state.

A controlled quantum scheme for teleporting an N-particle unknown state is investigated when N groups of three-particle W_1 states are used as quantum channels in this paper. Compared with Ref.[20], we adopt different measurement means and quantum channels. This scheme can reduce some of the difficulties in Refs.[17, 24–27]

The sender uses N groups of three-particle W_1 states as quantum channels shared with the recipient and the supervisors. After she makes Bell-state measurements on her particles, the sender publicizes her measurement results by a classical channel. If the supervisors agree to help the recipient restore the original state, they will perform the computational basis measurements on their particles, and inform the recipient of the measurement outcomes. Depending on the sender's and the supervisors' classical information, the recipient will introduce auxiliary particles and then perform unitary transformations on his particles and the auxiliary particles. In the end, the computational basis measurements will be performed by the recipient to confirm whether the teleportation succeeds or not. The successful completion of the scheme relies on all the supervisors' cooperation.

The rest of this paper is organized as follows. A controlled quantum teleportation scheme of N-particle unknown state is put forward in Section 2. In Section 3, we give some discussions in detail.

2. A controlled quantum teleportation scheme of an N-particle unknown state

In most references, the N-particle unknown state can be written in the following way:

$$|\Psi\rangle_N = x_0\underbrace{|0\ldots00\rangle}_{N}+x_1\underbrace{|0\ldots01\rangle}_{N} + \cdots + x_{2^N-1}\underbrace{|1\ldots11\rangle}_{N}. \tag{3}$$

This is complicated and troublesome for the author and the readers. Here, we apply a representation of the N-particle state in Ref.[15] which is concise in mathematical form. That is

$$|\Psi_0\rangle = \alpha_i\,|\Psi_i\rangle\,|0\rangle_i + \beta_i\,|\Psi_i'\rangle\,|1\rangle_i\,. \tag{4}$$

$|\Psi_0\rangle$ is the normalized wavefunction, where $|0\rangle_i$ and $|1\rangle_i$ are denoted as the orthogonal eigenstates of particle i which is selected at will. The quantum state of the remaining particles can be described as $|\Psi_i\rangle$ and $|\Psi_i'\rangle$ which are also normalized wavefunctions. The complex coefficients satisfy the normalized condition with $|\alpha_i|^2+|\beta_i|^2=1$. This representation is a general form of the N-particle state. It can be denoted as not only the N-particle entirely entangled state or the product states of one-particle state, but also the arbitrary combination of multi-particle entirely entangled state and multi-particle partially entangled state.

In order to realize the teleportation of the unknown state $|\Psi_0\rangle$, the sender (Alice) sets up the quantum channels shared with the recipient (Bob) and the supervisors (Charlie, David, *et al*), which are composed of N groups of three-particle W_1 states being expressed as the following equation:

$$|\Psi_W\rangle = \prod_{j=1}^{N}(a_j|100\rangle + b_j|010\rangle + c_j|001\rangle + d_j|111\rangle)_{A_jB_jC_j}\,, \tag{5}$$

where the coefficients are real and satisfy the normalized condition and are known to Bob. Furthermore, we assume the condition $a_j \geq b_j$ and $c_j \geq d_j$.

The N particles are in the unknown state $|\Psi_0\rangle$ to be teleported, particles A_j belong to Alice, particles B_j belong to Bob and particles C_j belong to the supervisors (Charlie, David, *et al*). Without loss of generality, we may study the teleporting process of arbitrary particle i. It is supposed that Charlie is the supervisor who is in control of particle C_i at this time.

The state of entire system includes N-particle unknown state and the quantum channels can be denoted as

$$\begin{aligned}|\Psi\rangle &= |\Psi_0\rangle|\Psi_W\rangle \\ &= \prod_{j=1}^{N}(a_j|100\rangle + b_j|010\rangle \\ &\quad + c_j|001\rangle + d_j|111\rangle)_{A_jB_jC_j} \\ &\quad \otimes(\alpha_i|\Psi\rangle|0\rangle_i + \beta_i|\Psi_i'\rangle|1\rangle_i).\end{aligned} \tag{6}$$

Eq.(6) can be rewritten as

$$
\begin{aligned}
|\Psi\rangle = & \prod_{j=1}^{N}{}'(a_j|100\rangle + b_j|010\rangle + c_j|001\rangle + d_j|111\rangle)_{A_jB_jC_j} \otimes [(\alpha_i|\Psi_i\rangle|0\rangle_i + \beta_i|\Psi_i'\rangle|1\rangle_i) \\
& \otimes(a_i|100\rangle + b_i|010\rangle + c_i|001\rangle + d_i|111\rangle)_{A_iB_iC_i}].
\end{aligned} \tag{7}
$$

Here $\prod_{j=1}^{N'}$ means successive multiplications in which j cannot equal i.

Let

$$
|\Phi_1\rangle = [(\alpha_i|\Psi_i\rangle|0\rangle_i + \beta_i|\Psi_i'\rangle|1\rangle_i) \otimes (a_i|100\rangle + b_i|010\rangle + c_i|001\rangle + d_i|111\rangle)_{A_iB_iC_i}]. \tag{8}
$$

In order to teleport state $|\Psi_0\rangle$, Alice performs Bell-state measurement on particle i and particle A_i, and then she informs Bob of her measurement outcome. The state $|\Phi_1\rangle$ will be projected into the following states:

$$
{}_{iA_i}\langle\Phi^{\pm}|\Phi_1\rangle = \frac{\sqrt{2}}{2}(b_i\alpha_i|\Psi_i\rangle|10\rangle_{B_iC_i} + c_i\alpha_i|\Psi_i\rangle|01\rangle_{B_iC_i} \pm a_i\beta_i|\Psi_i'\rangle|00\rangle_{B_iC_i} \pm d_i\beta_i|\Psi_i'\rangle|11\rangle_{B_iC_i}), \tag{9}
$$

$$
{}_{iA_i}\langle\Psi^{\pm}|\Phi_1\rangle = \frac{\sqrt{2}}{2}(a_i\alpha_i|\Psi_i\rangle|00\rangle_{B_iC_i} \pm b_i\beta_i|\Psi_i'\rangle|10\rangle_{B_iC_i} \pm c_i\beta_i|\Psi_i'\rangle|01\rangle_{B_iC_i} + d_i\alpha_i|\Psi_i\rangle|11\rangle_{B_iC_i}). \tag{10}
$$

Where,

$$
|\Phi^{\pm}\rangle_{iA_i} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{iA_i}, \qquad |\Psi^{\pm}\rangle_{iA_i} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{iA_i}. \tag{11}
$$

If Charlie agrees to help Bob restore the state which Alice wants to teleport, he performs the computational basis measurement on his particle C_i. Then, he informs Bob of his outcome through the classical channel. The possible outcomes are as follows:

$$
{}_{C_i}\langle 0|_{iA_i}\langle\Phi^{\pm}|\Phi_1\rangle = \frac{\sqrt{2}}{2}(\pm a_i\beta_i|\Psi_i'\rangle|0\rangle_{B_i} + b_i\alpha_i|\Psi_i\rangle|1\rangle_{B_i}), \tag{12}
$$

$$
{}_{C_i}\langle 1|_{iA_i}\langle\Phi^{\pm}|\Phi_1\rangle = \frac{\sqrt{2}}{2}(c_i\alpha_i|\Psi_i\rangle|0\rangle_{B_i} \pm d_i\beta_i|\Psi_i'\rangle|1\rangle_{B_i}), \tag{13}
$$

$$
{}_{C_i}\langle 0|_{iA_i}\langle\Psi^{\pm}|\Phi_1\rangle = \frac{\sqrt{2}}{2}(a_i\alpha_i|\Psi_i\rangle|0\rangle_{B_i} \pm b_i\beta_i|\Psi_i'\rangle|1\rangle_{B_i}), \tag{14}
$$

$$
{}_{C_i}\langle 1|_{iA_i}\langle\Psi^{\pm}|\Phi_1\rangle = \frac{\sqrt{2}}{2}(\pm c_i\beta_i|\Psi_i'\rangle|0\rangle_{B_i} + d_i\alpha_i|\Psi_i\rangle|1\rangle_{B_i}). \tag{15}
$$

If Bob wants to recover the state of particle i on particle B_i, he needs to introduce an auxiliary particle with $|0\rangle_{\mathrm{aux}_i}$ and perform unitary transformation U_i on particle B_i and particle aux_i. For example, when Eq.(12) is obtained, Bob should adopt unitary transformation which can be expressed as the following matrix:

$$
U_i = \begin{pmatrix}
0 & 1 & 0 & 0 \\
\pm\dfrac{b_i}{a_i} & 0 & 0 & \sqrt{1-\dfrac{b_i^2}{a_i^2}} \\
0 & 0 & 1 & 0 \\
\sqrt{1-\dfrac{b_i^2}{a_i^2}} & 0 & 0 & \mp\dfrac{b_i}{a_i}
\end{pmatrix}. \tag{16}
$$

$$
\begin{aligned}
& \frac{\sqrt{2}}{2}(\pm a_i\beta_i|\Psi_i'\rangle|00\rangle_{B_i\mathrm{aux}_i} + b_i\alpha_i|\Psi_i\rangle|10\rangle_{B_i\mathrm{aux}_i}) \\
\xrightarrow{U_i}\ & \frac{\sqrt{2}}{2}[b_i(\alpha_i|\Psi_i\rangle|0\rangle_{B_i} + \beta_i|\Psi_i'\rangle|1\rangle_{B_i})|0\rangle_{\mathrm{aux}_i} \\
& \pm(\sqrt{a_i^2-b_i^2}\beta_i|\Psi_i'\rangle|1\rangle_{B_i})|1\rangle_{\mathrm{aux}_i}].
\end{aligned} \tag{17}
$$

After the unitary transformation, Bob makes computational basis measurement on particle aux_i. If the outcome is $|0\rangle_{\mathrm{aux}_i}$, the step will be successfully realized with the probability $|b_i|^2$. The step fails when the result is $|1\rangle_{\mathrm{aux}_i}$.

The other three cases (when Eq.(13), or Eq.(14), or Eq.(15) is obtained) are similar to the above one, except that the unitary transformation requires a little modification. Accomplishing the above process, the information carried by particle i has been transferred to particle B_i and the probability of successful teleportation is $2(|b_i|^2 + |d_i|^2)$. The process can be described as in Fig.1.

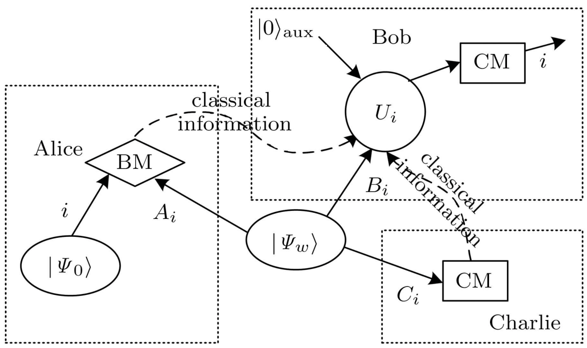

Fig.1. The process of teleporting the information of particle i. Firstly, Alice performs a Bell-state measurement (BM). Secondly, Charlie does a computational basis measurement (CM). Thirdly, Bob introduces an auxiliary particle and carries out a unitary transformation (U) depending on the classical information from Alice and Charlie. Finally, Bob makes a CM to confirm whether the teleportation succeeds or not.

The above process is only $1/N$ of the whole teleportation; the teleportation processes of the remaining particles are similar to particle i. When teleportation is ended, the state of particles in Bob's place can be written as

$$|\Psi_0\rangle_{\rm Bob} = \alpha_i |\Psi_i\rangle |1\rangle_{B_i} + \beta_i |\Psi_i'\rangle |0\rangle_{B_i} . \tag{18}$$

The state of Eq.(18) is formally equal to Eq.(5). That is to say, the original state belonging to Alice has replaced the state of particles in Bob's place. The N-particle unknown state has been successfully teleported to Bob. Because the choice of particle i is random, we can deduce that the total probability of successful teleportation is $2^N \prod_{j=1}^{N} (|b_j|^2 + |d_j|^2)$.

The scheme can also be realized in the following way: Alice and the supervisors complete all the measurements and inform Bob of all the outcomes. In succession, Bob introduces auxiliary particles and performs unitary transformations and computational basis measurements.

Let us now discuss the fidelity over the process of the transmission of the information of particle i.

(1) Consider the case that Charlie does not collaborate with Bob.

Let ρ_0 represents the density operator of the initial state of particle i. It is obvious that

$$\rho_0 = |\alpha_i|^2 |0\rangle\langle 0| + |\beta_i|^2 |1\rangle\langle 1| . \tag{19}$$

Let ρ_{Bj} represents the density operator of the state of particle B_i when the outcome of Alice's Bell-state measurement is $|R_j\rangle$. Here

$$|R_j\rangle = \{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle\} \quad (j = 1, 2, 3, 4). \tag{20}$$

According to the Refs.[11,30], the fidelity is

$$F_{Bj} = {\rm tr}(\rho_0 \rho_{Bj}). \tag{21}$$

The fidelity averaged over all outcomes $\{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle\}$ can be written as

$$F_{{\rm ave},B} = \sum_{j=1}^{4} P_j F_{Bj}, \tag{22}$$

where P_j is the probability for the outcome $|R_j\rangle$, so

$$F_{{\rm ave},B} = a_i^2 |\alpha_i|^2 + b_i^2 |\alpha_i|^2 + c_i^2 |\beta_i|^2 + d_i^2 |\beta_i|^2 . \tag{23}$$

Similarly,

$$\begin{aligned} F_{{\rm ave},C} &= \sum_{j=1}^{4} P_j F_{Cj} \\ &= a_i^2 |\alpha_i|^2 + b_i^2 |\beta_i|^2 + c_i^2 |\alpha_i|^2 + d_i^2 |\beta_i|^2 , \end{aligned} \tag{24}$$

where $F_{Cj} = {\rm tr}(\rho_0 \rho_{Cj})$, ρ_{Cj} is the density operator of the state of particle C_i when the outcome of Alice's Bell-state measurement is $|R_j\rangle$. Especially, when $a_i = b_i = c_i = d_i = 1/2$, the fidelities become

$$F_{{\rm ave},B} = F_{{\rm ave},C} = 1/2. \tag{25}$$

It should be noted that the fidelity $F = 1/2$ corresponds to a random result. In other words, Bob cannot get the teleported state without the cooperation of Charlie, and Charlie cannot get the state himself as well.

(2) Consider the case that Charlie collaborates with Bob.

$$F_{{\rm ave},B} = \sum_{k=1}^{2} \sum_{j=1}^{4} P_{kj} F_{Bkj}, \tag{26}$$

where $F_{Bkj} = {\rm tr}(\rho_0 \rho_{Bkj})$, ρ_{Bkj} and P_{kj} correspond to the density operator of the state of particle B_i and the probability that Alice's Bell-state measurement outcome is $|R_j\rangle$ and Charlie's computational basis measurement is $|Q_k\rangle$ respectively, where

$$|Q_k\rangle = \{|0\rangle, |1\rangle\}, (k=1,2). \tag{27}$$

So

$$F_{\mathrm{ave},B} = 1 - 2(a_i^2 + c_i^2 - b_i^2 - d_i^2)\,|\alpha_i\beta_i|^2, \tag{28}$$

$$F_{\mathrm{ave},B} = 1 \quad \text{for} \quad a_i = b_i = c_i = d_i = 1/2. \tag{29}$$

3. Discussions

For the security of teleportation, we may adopt a similar way as used in the protocol in Ref.[31]. That is, Alice produces many groups of three-particle W_1 state which can be expressed as Eq.(2). They are divided into three sequences. That is, A sequence (particles A_j), B sequence (particles B_j), and C sequence (particles C_j). She transmits B sequence and C sequence to Bob and the supervisors respectively. Then, they measure a subset of particles in their hands. After that, they analyse the security of the teleportation. If the error rate of the sampling particles is reasonably low, Alice, Bob, and supervisors can trust the quantum channel, and continue with the next processing. Otherwise, they abandon the processing.

In the scheme, Bob can recover the state from Alice with all the supervisors' cooperation. If any one of the supervisors doesn't agree to cooperate, Bob has no way to obtain the state that Alice wants to transmit. From Eq.(12) to Eq.(15), we can see that Bob has only the probability 1/2 to choose the correct local unitary transformation for reconstructing the state of particle i if he knows the classical information from Alice after Charlie performed the computational basis measurement. If Charlie doesn't perform the measurement, Bob can only obtain a random outcome, and no useful information about the state of particle i. For the same result, Charlie cannot acquire the state of particle i as Bob does not perform the measurement and cooperate with him. So the recipient cannot acquire the state to be teleported without the supervisors' cooperation and the supervisors cannot get the information of the original state as well.

The manipulation order of measurements of the sender and the supervisors may be exchanged in this scheme. The different supervisors can make measurements on their own particle after receiving the information when teleportation begins if they agree to cooperate. They may perform at the same time or in turn. These ways do not affect the probability of successful teleportation. The state to be teleported can be transmitted to N supervisors uniformly if the recipient agrees to do it.

The recipient and the supervisor may exchange roles if there is only one supervisor in the process. When the coefficients $a_i = b_i = c_i = d_i = 1/2$, Bob need not introduce the auxiliary particle and may only perform the single qubit unitary transformation. Under this condition, the intrinsic efficiency for qubits η_q in this scheme approaches one, the same as the result in Ref.[20].

In summary, in this paper a controlled quantum scheme for teleporting an N-particle unknown state is investigated when N groups of three-particle W_1 states are used as quantum channels. The advantages of this scheme are as follows: the sender and the supervisors may perform their own operations in random sequence, and the methods of measurement and unitary transformations in experiment are simple and easy to fulfil. So the scheme is flexible and feasible in reality.

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